Electromagnetic Dyadic Green's Function in Cylindrically Multilayered Media

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Abstract—**A spectral-domain dyadic Green's function for electromagnetic fields in cylindrically multilayered media with circular cross section is derived in terms of matrices of the cylindrical vector wave functions. Some useful concepts, such as the effective plane wave reflection and transmission coefficients, are extended in the present spectral domain eigenfunction expansion. The coupling coefficient matrices of the scattering dyadic Green's functions are given by applying the principle of scattering superposition. The general solution has been applied to the case of axial symmetry (n =0, n is eigenvalue parameter in $\hat{\phi}$ direction) where the scattering coefficients are decoupled between TM and TE waves. Two specific geometries, i.e., two- and three-layered media that are frequently employed to model the practical problems are considered in detail, and the coupling coefficient matrices of their dyadic Green's functions are given, respectively.**

I. Introduction

THE dyadic Green's function (DGF) technique has been widely used to investigate the electromagnetic waves [1]–[5] for more than 20 years. Although only in a few simple geometries can the dyadic Green's functions be obtained in closed form, the compact formulations and solutions of some electromagnetic problems they offer make their use extremely attractive. The dyadic Green's functions of canonical problems may be constructed in several ways. One of the most common approaches is to express the Green's functions in terms of a magnetic vector potential [5]–[8], whereas another approach is to construct the Green's function from a set of appropriate electric and magnetic vector potentials [9]–[11]. Of the constructing approaches, the vector wave function (VWF) approach is more widely employed to generate dyadic Green's functions [1]–[4], [12], [13].

Many scholars are interested in the research on the dyadic Green's functions in layered media. Among the research work, the dyadic Green's function constructed for planarly stratified media is the most well developed at present. The planarly multilayered media, such as the planarly stratified and isotropic medium [1]–[4], [14], [15], the slow-variation multilayered medium [16], the multilayered anisotropic medium [17], and the multilayered chiral medium investigated recently [18] have been applied by many researchers to model the physical geometries of practical engineering problems.

The dyadic Green's functions for spherically multilayered media have also received increased interest. The DGF's for some simple spherical geometry and for the inhomogeneous spherical lenses were presented earlier in [1], [4]. More recently, the investigation of the dyadic Green's function has been extended to media of more complicated spherical multilayered geometries [19]–[22].

The electromagnetic waves in cylindrically multilayered media have been investigated by many researchers [1]–[5], [23]–[25]. Unlike the waves in the planarly and spherically stratified media, the waves in the cylindrically multilayered media are usually coupled not only between TM modes and between TE modes, but also between TM and TE modes. Although the dyadic Green's function for cylindrically multilayered media was investigated by some researchers to a great extent [1], [2], [4], [23], [24], [26], the general expression of the dyadic Green's function for the electromagnetic fields in an arbitrary multilayered media with circular cross section has not been reported except under the condition that the problems under consideration are either rotationally symmetric ($n = 0$) or when the field is a two-dimensional one, being independent of the longitudinal axis ($\partial/\partial z = 0$). Many complex practical problems require a large number of layers to model the propagation environment and/or scatters. For example, in optical fibers the doping of the fiber may have a gradual transition rather than a step transition. Such a gradual transition may be modeled with many thin layers of piecewise homogeneous layers. In the geophysical exploration of the subsurface earth, a bore hole drilled deep into the earth is often employed. Such bore holes are usually filled with fluid. The subsequent invasion of bore-hole fluid into the rock formation gives rise to an altered zone whose electromagnetic property varies radially away from the bore hole. In reducing the radar scattering cross section by the inlets of aircrafts, we usually coat the inlets with multilayered materials to improve and control the scattering properties [25], [27]. Such kinds of practical engineering problems may be modeled by a cylindrically stratified media. It will be very useful, therefore, to produce a general expression of dyadic Green's function in a cylindrically multilayered media with circular cross section.

In this paper, a rigorous formulation of dyadic Green's function for the problem of electromagnetic radiation from a point source of excitation embedded in an arbitrary layer of the circular cylindrically multilayered media is presented in terms of matrices of the cylindrical VWF. Some useful concepts such as the effective plane wave reflection and transmission coefficients are extended in the present spectral domain eigenfunction expansion. The recursive matrices of coupling coefficients for the scattering dyadic Green's functions are derived by applying the principle of scattering superposition. The

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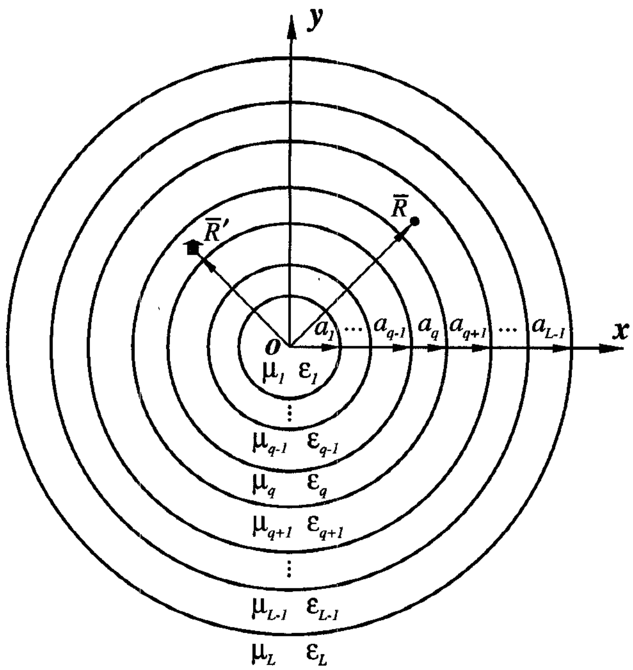

Fig. 1. Geometry of a cylindrical multilayered medium.

general solution has been applied to the case of axial symmetry ($n = 0$) where the scattering coefficients are decoupled. The other two specific cases of two- and three-layered media are also considered in particular as examples, assuming the current source is located in various layers of the medium.

II. Formulation of the Problem

Consider a circular cylindrically multilayered lossy or lossless medium of L layers, as shown in Fig. 1. The electromagnetic radiation fields, $\boldsymbol{E}_p$ and $\boldsymbol{H}_p$ in the pth layer ($p = 1, 2, \cdots, L$), contributed by the electric and magnetic current sources $\boldsymbol{J}_q$ and $\boldsymbol{M}_q$ lying in the qth layer ($q = 1, 2, \cdots, L$), are given by

$$\nabla \times \nabla \times \boldsymbol{E}_p - k_p^2 \boldsymbol{E}_p = i\omega\mu_p \boldsymbol{J}_p \delta_p^q - (\nabla \times \boldsymbol{M}_p)\delta_p^q \quad (1)$$

$$\nabla \times \nabla \times \boldsymbol{H}_p - k_p^2 \boldsymbol{H}_p = i\omega\epsilon_p \boldsymbol{M}_p \delta_p^q + (\nabla \times \boldsymbol{J}_p)\delta_p^q \quad (2)$$

where ϵ_p and μ_p are the complex permittivity and permeability, and $k_p = \omega\sqrt{\mu_p \epsilon_p}$ is the wave number in the lossy or lossless medium of the pth stratified layer, respectively. δ_p^q stands for the Kronecker delta. A time dependence $\exp(-i\omega t)$ is assumed for the fields throughout this paper.

According to the duality and the superposition of electromagnetic fields, only the electric type of dyadic Green's function due to an electric current source needs to be solved in order to avoid unnecessary repetition. The magnetic type of dyadic Green's function can be easily obtained according to duality principle. The general formulation for the electric field $\boldsymbol{E}_p$ excited by an electric current source $\boldsymbol{J}_q$ can be given by

$$\boldsymbol{E}_p(\overline{R}) = i\omega\mu_p \iiint_{V_q} \overline{\overline{G}}_e^{(pq)}(\overline{R}, \overline{R}') \cdot \boldsymbol{J}_q(\overline{R}')\, dV' \quad (3)$$

where the V_q is the volume occupied by the sources embedded in the qth layer, $\overline{R}$ stands for the field point, (r, ϕ, z), and $\overline{R}'$ the source point, (r', ϕ', z'). Substituting (3) into (1) yields

$$\nabla \times \nabla \times \overline{\overline{G}}_e^{(pq)}(\overline{R}, \overline{R}') - k_p^2 \overline{\overline{G}}_e^{(pq)}(\overline{R}, \overline{R}') = \overline{\overline{I}}\delta(\overline{R} - \overline{R}')\delta_p^q \quad (4)$$

where $\overline{\overline{I}}$ is the unit dyadic and $\delta(\overline{R} - \overline{R}')$ the Dirac delta function. The electric type of dyadic Green's function $\overline{\overline{G}}_e^{(pq)}(\overline{R}, \overline{R}')$ satisfies the following boundary conditions at the circular cylindrical interfaces $r = a_l (l = 1, 2, \cdots, L-1)$

$$\hat{r} \times \overline{\overline{G}}_e^{(pq)} = \hat{r} \times \overline{\overline{G}}_e^{[(p+1)q]} \quad (5)$$

$$\frac{1}{\mu_p}\hat{r} \times \nabla \times \overline{\overline{G}}_e^{(pq)} = \frac{1}{\mu_{p+1}}\hat{r} \times \nabla \times \overline{\overline{G}}_e^{[(p+1)q]}. \quad (6)$$

The magnetic fields $\boldsymbol{H}_p$ due to the excitation of electric current source $\boldsymbol{J}_q$ can be found from Maxwell's functions as

$$\boldsymbol{H}_p(\overline{R}) = \iiint_{V_q} \nabla \times \overline{\overline{G}}_e^{(pq)}(\overline{R}, \overline{R}') \cdot \boldsymbol{J}_q(\overline{R}')\, dV'. \quad (7)$$

The solutions to the electromagnetic fields due to the excitation of a magnetic current distribution $\boldsymbol{M}_q$ can be easily obtained by following the similar procedure as due to the excitation of an electric current source $\boldsymbol{J}_q$. Therefore, only the electric type of dyadic Green's function will be presented in this paper.

III. General Expression of Dyadic Green's Function

As mentioned above, several approaches may be employed to construct the electromagnetic dyadic Green's functions in the stratified media. In this paper, we will use the vector wave function expansion to derive the dyadic Green's function in the circular cylindrically L-layered medium.

From (1) and/or (2), we can obtain the scalar eigenfunctions given by

$$\psi_{\substack{e\\o}\eta}(h) = Z_n(\eta r) \begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\exp(ihz) \quad (8)$$

where $Z_n(\eta r)$ is a general cylinder function (including $J_n, N_n, H_n^{(1)}$, and $H_n^{(2)}$) of order n, and η the propagation constant in r direction with $k^2 = \eta^2 + h^2$. The vector eigenfunctions can be constructed by

$$\boldsymbol{L}_{\substack{e\\o}\eta}(h) = \nabla\psi_{\substack{e\\o}\eta}(h) \quad (9)$$

$$\boldsymbol{M}_{\substack{e\\o}\eta}(h) = \nabla \times [\psi_{\substack{e\\o}\eta}(h)\hat{z}] \quad (10)$$

$$\boldsymbol{N}_{\substack{e\\o}\eta}(h) = \frac{1}{k}\nabla \times \nabla \times [\psi_{\substack{e\\o}\eta}(h)\hat{z}] \quad (11)$$

and the notation used here follows that in [1]. The group of equations given above can be written more explicitly using (8) as

$$\boldsymbol{L}_{\substack{e\\o}\eta}(h) = \left[\frac{dZ_n(\eta r)}{dr}\begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\hat{r} \mp \frac{nZ_n(\eta r)}{r}\begin{matrix}\sin\\ \cos\end{matrix}(n\phi)\hat{\phi} + ihZ_n(\eta r)\begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\hat{z}\right]\exp(ihz) \quad (12)$$

$$\boldsymbol{M}_{\substack{e\\o}\eta}(h) = \left[\mp\frac{nZ_n(\eta r)}{r}\begin{matrix}\sin\\ \cos\end{matrix}(n\phi)\hat{r} - \frac{dZ_n(\eta r)}{r}\begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\hat{\phi}\right]\exp(ihz) \quad (13)$$

$$\boldsymbol{N}_{\substack{e\\o}\eta}(h)=\frac{1}{k}\left[ih\frac{dZ_n(\eta r)}{dr}\begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\hat{r}\mp\frac{ihn}{r}Z_n(\eta r)\begin{matrix}\sin\\ \cos\end{matrix}(n\phi)\hat{\phi}\right.$$
$$\left.+\ \eta^2 Z_n(\eta r)\begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\hat{z}\right]\exp(ihz). \tag{14}$$

For the geometry shown in Fig. 1, an incoming wave is always totally reflected at the origin resulting in standing wave so that there exist only two types of waves—standing wave and outgoing wave, which can be represented by J_n and $H_n^{(1)}$, respectively. The vector wave functions in (12)–(14) have been verified [1] to be orthogonal among themselves as well as with respect to each other, as they are integrated over all the values of r, ϕ, and z. The expansion of vector wave functions in terms of J_n and $H_n^{(1)}$ functions has actual physical meaning and is advantageous compared with the usual expansion in orthogonal function pairs (J_n, N_n) or $(H_n^{(1)}, H_n^{(2)})$. We can use those cylindrical vector wave functions to construct the desired dyadic Green's function.

According to the principle of scattering superposition, the dyadic Green's function can be considered as the superposition of $\overline{\overline{G}}_{e0}(\overline{R},\overline{R}')$ (an unbounded DGF) and $\overline{\overline{G}}_{es}^{(pq)}(\overline{R},\overline{R}')$ (a scattering DGF). $\overline{\overline{G}}_{e0}(\overline{R},\overline{R}')$ corresponds to the contribution of the direct waves from radiation sources in the infinite homogeneous space and $\overline{\overline{G}}_{es}^{(pq)}(\overline{R},\overline{R}')$ to the additional contribution of the multiple reflection and transmission waves due to the presence of dielectric interfaces. The complete dyadic Green's function, $\overline{\overline{G}}_{e}^{(pq)}(\overline{R},\overline{R}')$, is therefore given by

$$\overline{\overline{G}}_{e}^{(pq)}(\overline{R},\overline{R}')=\overline{\overline{G}}_{e0}(\overline{R},\overline{R}')\delta_p^q+\overline{\overline{G}}_{es}^{(pq)}(\overline{R},\overline{R}') \tag{15}$$

where the superscript (pq) denotes the layers in which the field point $\overline{R}$ and source point $\overline{R}'$ locate, respectively, and the subscript s means the scattering dyadic Green's functions.

The dyadic Green's function $\overline{\overline{G}}_{e0}(\overline{R},\overline{R}')$ in the unbounded medium is given in Tai's well-known book [1] by using the contour integration in the complex η-plane as a result of the residue theorem, as follows:

$$\overline{\overline{G}}_{e0}(\overline{R},\overline{R}')=\frac{\hat{r}\hat{r}}{k^2}\delta(\overline{R}-\overline{R}')+\frac{i}{8\pi}\int_{-\infty}^{+\infty}dh\sum_{n=0}^{\infty}\frac{2-\delta_0}{\eta^2}$$
$$\times\begin{cases}\boldsymbol{M}_{\substack{e\\o}\eta}^{(1)}\boldsymbol{M}'_{\substack{e\\o}\eta}+\boldsymbol{N}_{\substack{e\\o}\eta}^{(1)}\boldsymbol{N}'_{\substack{e\\o}\eta}, & \text{if } r>r'\\ \boldsymbol{M}_{\substack{e\\o}\eta}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta}+\boldsymbol{N}_{\substack{e\\o}\eta}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta}, & \text{if } r<r'\end{cases} \tag{16}$$

where simplified notations (throughout the paper) $\boldsymbol{M}_{\substack{e\\o}\eta}=\boldsymbol{M}_{\substack{e\\o}\eta}(h), \boldsymbol{M}_{\substack{e\\o}\eta}^{(1)}=\boldsymbol{M}_{\substack{e\\o}\eta}^{(1)}(h), \boldsymbol{M}'_{\substack{e\\o}\eta}=\boldsymbol{M}'_{\substack{e\\o}\eta}(-h)$, and $\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta}=\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta}(-h)$ stand for the electric field of the TE mode, while $\boldsymbol{N}_{\substack{e\\o}\eta}=\boldsymbol{N}_{\substack{e\\o}\eta}(h), \boldsymbol{N}_{\substack{e\\o}\eta}^{(1)}=\boldsymbol{N}_{\substack{e\\o}\eta}^{(1)}(h), \boldsymbol{N}'_{\substack{e\\o}\eta}=\boldsymbol{N}'_{\substack{e\\o}\eta}(-h)$, and $\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta}=\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta}(-h)$ represent that of the TM mode. The prime denotes the coordinates (r', ϕ', z') of the current source $\boldsymbol{J}(\overline{R}'), n$ identifies the eigenvalue parameter, and δ_0 the Kronecker delta function defined with respect to n. $H_n^{(1)}$ should be chosen for Z_n if vector wave function has the superscript (1); otherwise, J_n should be chosen for Z_n in the expression of the cylindrical vector wave functions.

As discussed above, the electromagnetic fields in the geometry we consider consist of the radial standing wave modes and outgoing wave modes. It is this important feature that facilitates greatly to simplify the eigenfunction expansion of the new dyadic Green's function by using the cylinder vector wave functions only in terms of J_n and $H_n^{(1)}$ functions. Assuming that the current source is located in the qth layer, we may construct the scattering dyadic Green's function for the pth layer $(p = 1, 2, \cdots, L)$ as follows:

$$\overline{\overline{G}}_{es}^{(pq)}(\overline{R},\overline{R}')$$
$$=\frac{i}{8\pi}\int_{-\infty}^{+\infty}dh\sum_{-\infty}^{\infty}\frac{2-\delta_0}{\eta_p^2}$$
$$\times\{[\boldsymbol{N}_{\substack{e\\o}\eta_p},\boldsymbol{M}_{\substack{e\\o}\eta_p}]\cdot\overline{\mathcal{A}}_{n(00)}^{(pq)}\cdot[\boldsymbol{N}'_{\substack{e\\o}\eta_q},\boldsymbol{M}'_{\substack{e\\o}\eta_q}]^T$$
$$+[\boldsymbol{N}_{\substack{e\\o}\eta_p},\boldsymbol{M}_{\substack{e\\o}\eta_p}]\cdot\overline{\mathcal{B}}_{n(01)}^{(pq)}\cdot[\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_q},\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_q}]^T$$
$$+[\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_p},\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_p}]\cdot\overline{\mathcal{C}}_{n(10)}^{(pq)}\cdot[\boldsymbol{N}'_{\substack{e\\o}\eta_q},\boldsymbol{M}'_{\substack{e\\o}\eta_q}]^T$$
$$+[\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_p},\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_p}]\cdot\overline{\mathcal{D}}_{n(11)}^{(pq)}\cdot[\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_q},\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_q}]^T\} \tag{17}$$

where $\overline{\mathcal{A}}_{n(00)}^{(pq)}, \overline{\mathcal{B}}_{n(01)}^{(pq)}, \overline{\mathcal{C}}_{n(10)}^{(pq)}$, and $\overline{\mathcal{D}}_{n(11)}^{(pq)} (p, q = 1, 2, \cdots, L)$ are the matrices of order 2 × 2 and stand for the coefficient matrices of the scattering dyadic Green's function to be solved. The subscript n means that all of them are the functions with respect to n, and the subscripts (00), (01), (10), and (11) describe the characteristics that the left-side number and the right-side number stand for the operation on the field and the source coordinates, respectively. Moreover, the numbers 0 and 1 require that the Bessel and Hankel functions of the first kind, J_n and $H_n^{(1)}$, should be chosen for the cylinder function Z_n in the vector wave functions, respectively. The superscript T denotes the transposition of matrices and L the number of the layers of the cylindrical medium. Those coefficient matrices of order 2 × 2 can be given more explicitly by

$$\overline{\mathcal{A}}_{n(00)}^{(pq)}=\begin{pmatrix}A_{NN}^{(pq)} & A_{NM}^{(pq)}\\ A_{MN}^{(pq)} & A_{MM}^{(pq)}\end{pmatrix},\quad \overline{\mathcal{B}}_{n(01)}^{(pq)}=\begin{pmatrix}B_{NN}^{(pq)} & B_{NM}^{(pq)}\\ B_{MN}^{(pq)} & B_{MM}^{(pq)}\end{pmatrix}$$
$$\overline{\mathcal{C}}_{n(10)}^{(pq)}=\begin{pmatrix}C_{NN}^{(pq)} & C_{NM}^{(pq)}\\ C_{MN}^{(pq)} & C_{MM}^{(pq)}\end{pmatrix},\quad \overline{\mathcal{D}}_{n(11)}^{(pq)}=\begin{pmatrix}D_{NN}^{(pq)} & D_{NM}^{(pq)}\\ D_{MN}^{(pq)} & D_{MM}^{(pq)}\end{pmatrix} \tag{18}$$

where all the elements in the matrices above stand for the scattering coefficients of the scattering dyadic Green's function to be solved. The diagonal terms of the matrices above denote the self-coupling of the wave, that is, TM to TM or TE to TE coupling. But the off-diagonal terms of those matrices denote the coupling from the TE to TM, and the TM to TE waves.

All the coefficient matrices of the dyadic Green's function can be defined from the boundary conditions given in (5) and (6). In order to avoid more complicated operation, some concepts of the effective plane wave reflection and transmission are applied and extended for our problems. The final solutions to those scattering coefficient matrices of the dyadic Green's function can be derived as follows:

$$\overline{\mathcal{A}}_{n(00)}^{(pq)}=\tilde{\overline{R}}_{p,p+1}\cdot(\overline{I}-\tilde{\overline{R}}_{p,p-1}\cdot\tilde{\overline{R}}_{p,p+1})^{-1}\cdot\tilde{\overline{T}}_{qp}$$
$$\cdot(\overline{I}-\tilde{\overline{R}}_{q,q+1}\cdot\tilde{\overline{R}}_{q,q-1})^{-1}U(p-q)$$

$$+ (\overline{I} - \tilde{\overline{R}}_{p,p+1} \cdot \tilde{\overline{R}}_{p,p-1})^{-1} \cdot \tilde{\overline{R}}_{p,p+1} \delta_p^q$$
$$+ (\overline{I} - \tilde{\overline{R}}_{p,p-1} \cdot \tilde{\overline{R}}_{p,p+1})^{-1} \cdot \tilde{\overline{T}}_{qp}$$
$$\cdot (\overline{I} - \tilde{\overline{R}}_{q,q+1} \cdot \tilde{\overline{R}}_{q,q-1})^{-1} \cdot \tilde{\overline{R}}_{q,q+1} U(q-p) \quad (19)$$

$$\overline{\mathcal{B}}_{n(01)}^{(pq)} = \tilde{\overline{R}}_{p,p+1} \cdot (\overline{I} - \tilde{\overline{R}}_{p,p-1} \cdot \tilde{\overline{R}}_{p,p+1})^{-1} \cdot \tilde{\overline{T}}_{qp}$$
$$\cdot (\overline{I} - \tilde{\overline{R}}_{q,q-1} \cdot \tilde{\overline{R}}_{q,q+1})^{-1} \cdot \tilde{\overline{R}}_{q,q-1} U(p-q)$$
$$+ (\overline{I} - \tilde{\overline{R}}_{p,p+1} \cdot \tilde{\overline{R}}_{p,p-1})^{-1} \cdot \tilde{\overline{R}}_{q,q+1} \cdot \tilde{\overline{R}}_{q,q-1} \delta_p^q$$
$$+ (\overline{I} - \tilde{\overline{R}}_{p,p-1} \cdot \tilde{\overline{R}}_{p,p+1})^{-1} \cdot \tilde{\overline{T}}_{qp}$$
$$\cdot (\overline{I} - \tilde{\overline{R}}_{q,q-1} \cdot \tilde{\overline{R}}_{q,q+1})^{-1} U(q-p) \quad (20)$$

$$\overline{\mathcal{C}}_{n(10)}^{(pq)} = (\overline{I} - \tilde{\overline{R}}_{p,p+1} \cdot \tilde{\overline{R}}_{p,p-1})^{-1} \cdot \tilde{\overline{T}}_{qp}$$
$$\cdot (\overline{I} - \tilde{\overline{R}}_{q,q+1} \cdot \tilde{\overline{R}}_{q,q-1})^{-1} U(p-q)$$
$$+ (\overline{I} - \tilde{\overline{R}}_{p,p-1} \cdot \tilde{\overline{R}}_{p,p+1})^{-1} \cdot \tilde{\overline{R}}_{p,p-1} \cdot \tilde{\overline{R}}_{p,p+1} \delta_p^q$$
$$+ \tilde{\overline{R}}_{p,p-1} \cdot (\overline{I} - \tilde{\overline{R}}_{p,p+1} \cdot \tilde{\overline{R}}_{p,p-1})^{-1} \cdot \tilde{\overline{T}}_{qp}$$
$$\cdot (\overline{I} - \tilde{\overline{R}}_{q,q+1} \cdot \tilde{\overline{R}}_{q,q-1})^{-1} \cdot \tilde{\overline{R}}_{q,q+1} U(q-p) \quad (21)$$

$$\overline{\mathcal{D}}_{n(11)}^{(pq)} = (\overline{I} - \tilde{\overline{R}}_{p,p+1} \cdot \tilde{\overline{R}}_{p,p-1})^{-1} \cdot \tilde{\overline{T}}_{qp}$$
$$\cdot (\overline{I} - \tilde{\overline{R}}_{q,q-1} \cdot \tilde{\overline{R}}_{q,q+1})^{-1} \cdot \tilde{\overline{R}}_{q,q-1} U(p-q)$$
$$+ (\overline{I} - \tilde{\overline{R}}_{p,p-1} \cdot \tilde{\overline{R}}_{p,p+1})^{-1} \cdot \tilde{\overline{R}}_{p,p-1} \delta_p^q$$
$$+ \tilde{\overline{R}}_{p,p-1} \cdot (\overline{I} - \tilde{\overline{R}}_{p,p+1} \cdot \tilde{\overline{R}}_{p,p-1})^{-1} \cdot \tilde{\overline{T}}_{qp}$$
$$\cdot (\overline{I} - \tilde{\overline{R}}_{q,q-1} \cdot \tilde{\overline{R}}_{q,q+1})^{-1} U(q-p) \quad (22)$$

where the step function $U(x)$ is defined by

$$U(x) = \begin{cases} 1, & x > 0, \\ 0, & x \le 0. \end{cases} \quad (23)$$

$\tilde{\overline{R}}$ and $\tilde{\overline{T}}$ are the generalized reflection and transmission coefficient matrices, respectively, and can be derived by a set of recursive formulations as follows:

$$\tilde{\overline{R}}_{p,p-1} = \overline{R}_{p,p-1} + \overline{T}_{p-1,p} \cdot \tilde{\overline{R}}_{p-1,p-2} \cdot (\overline{I} - \overline{R}_{p-1,p} \cdot \tilde{\overline{R}}_{p-1,p-2})^{-1} \cdot \overline{T}_{p,p-1} \quad (24)$$

$$\tilde{\overline{R}}_{p,p+1} = \overline{R}_{p,p+1} + \overline{T}_{p+1,p} \cdot \tilde{\overline{R}}_{p+1,p+2} \cdot (\overline{I} - \overline{R}_{p+1,p} \cdot \tilde{\overline{R}}_{p+1,p+2})^{-1} \cdot \overline{T}_{p,p+1} \quad (25)$$

$$\tilde{\overline{T}}_{1L} = \overline{Q}_{L-1,L} \cdot \overline{Q}_{L-2,L-1} \cdots \overline{Q}_{12} \quad (26)$$

$$\tilde{\overline{T}}_{L1} = \overline{Q}_{L,L-1} \cdot \overline{Q}_{L-1,L-2} \cdots \overline{Q}_{21} \quad (27)$$

$$\overline{Q}_{p,p+1} = (\overline{I} - \overline{R}_{p+1,p} \cdot \tilde{\overline{R}}_{p+1,p+2})^{-1} \cdot \overline{T}_{p,p+1} \quad (28)$$

$$\overline{Q}_{p,p-1} = (\overline{I} - \overline{R}_{p-1,p} \cdot \tilde{\overline{R}}_{p-1,p-2})^{-1} \cdot \overline{T}_{p,p-1} \quad (29)$$

where $p = 1, 2, \cdots, L$. Note that $\tilde{\overline{R}}_{L,L+1} = 0$, and $\tilde{\overline{R}}_{1,0} = 0$. The $\overline{R}$ and $\overline{T}$ are the local reflection and transmission coefficient matrices. At the pth interface of the medium, those 2×2 coefficient matrices are given by

$$\overline{R}_{p,p+1} = [\overline{J}_n(\eta_p a_p) H_n^{(1)}(\eta_{p+1} a_p) - \overline{H}_n^{(1)}(\eta_{p+1} a_p) J_n(\eta_p a_p)]^{-1} \cdot [\overline{H}_n^{(1)}(\eta_{p+1} a_p) H_n^{(1)}(\eta_p a_p) - \overline{H}_n^{(1)}(\eta_p a_p) H_n^{(1)}(\eta_{p+1} a_p)] \quad (30)$$

$$\overline{R}_{p+1,p} = [\overline{J}_n(\eta_p a_p) H_n^{(1)}(\eta_{p+1} a_p) - \overline{H}_n^{(1)}(\eta_{p+1} a_p) J_n(\eta_p a_p)]^{-1} \cdot [\overline{J}_n(\eta_{p+1} a_p) J_n(\eta_p a_p) - \overline{J}_n(\eta_p a_p) J_n(\eta_{p+1} a_p)] \quad (31)$$

$$\overline{T}_{p,p+1} = \frac{2\omega}{\pi \eta_p^2 a_p} [\overline{J}_n(\eta_p a_p) H_n^{(1)}(\eta_{p+1} a_p) - \overline{H}_n^{(1)}(\eta_{p+1} a_p) J_n(\eta_p a_p)]^{-1} \cdot \begin{pmatrix} \epsilon_p & 0 \\ 0 & -\mu_p \end{pmatrix} \quad (32)$$

$$\overline{T}_{p+1,p} = \frac{2\omega}{\pi \eta_{p+1}^2 a_p} [\overline{J}_n(\eta_p a_p) H_n^{(1)}(\eta_{p+1} a_p) - \overline{H}_n^{(1)}(\eta_{p+1} a_p) J_n(\eta_p a_p)]^{-1} \cdot \begin{pmatrix} \epsilon_{p+1} & 0 \\ 0 & -\mu_{p+1} \end{pmatrix} \quad (33)$$

where $a_l (l = 1, 2, \cdots, L-1)$ denotes the radius of the lth layer, and the $\overline{J}_n$ and $\overline{H}_n$ the 2×2 matrices defined in the form as follows:

$$\overline{B}_n(\eta_p r) = \frac{1}{\eta_p^2 r} \begin{bmatrix} i\omega\epsilon_p \eta_p r B_n'(\eta_p r) & -nhB_n(\eta_p r) \\ -nhB_n(\eta_p r) & -i\omega\mu_p \eta_p r B_n'(\eta_p r) \end{bmatrix} \quad (34)$$

where B_n is either $H_n^{(1)}$ or J_n depending on whether we are defining the $\overline{H}_n^{(1)}$ matrix or the $\overline{J}_n$ matrix, and the prime, the derivative, operates with respect to the whole variable of the relevant functions. Moreover, $\overline{B}_n$ is diagonal when $n = 0$.

So far, the general expressions of the scattering dyadic Green's functions for the cylindrical arbitrary multilayered media with circular cross section have been obtained. In order to show how to simplify the general expressions for some specific cases, we will consider the dyadic Green's functions for the the case of axial symmetry ($n = 0$), as well as two specific geometries, i.e., two- and three-layered media that are frequently employed to model the practical problems.

IV. Specific DGF's for Several Special Cases

The general dyadic Green's function in cylindrically multilayered media is given by (15). The mathematical expression of unbounded DGF, $\overline{\overline{G}}_{e0}(\overline{R}, \overline{R}')$, remains unchanged for the layer where the source is located only if we replace the propagation constant k and the radial propagation constant η in (16) by those of the source layer. The dyadic Green's functions can be easily given if the scattering term $\overline{\overline{G}}_{es}^{(pq)}(\overline{R}, \overline{R}')$ is obtained. Therefore, we consider only the scattering dyadic Green's functions, as well as their coefficients or coefficient matrices, for those special cases.

A. The Case of Axial Symmetry ($n = 0$)

From the fact that $\overline{B}_n$ is diagonal when $n = 0$, we can easily know that all the reflection and transmission matrices, $\overline{R}, \tilde{\overline{R}}, \overline{T}$ and $\tilde{\overline{T}}$, are diagonal. Furthermore, $\overline{\mathcal{A}}_{n(00)}^{(pq)}, \overline{\mathcal{B}}_{n(01)}^{(pq)}, \overline{\mathcal{C}}_{n(10)}^{(pq)}$ and $\overline{\mathcal{D}}_{n(11)}^{(pq)}$ are also diagonal, that is, $A_{NM}^{(pq)}, A_{MN}^{(pq)}, B_{NM}^{(pq)}, B_{MN}^{(pq)}, C_{NM}^{(pq)}, C_{MN}^{(pq)}, D_{NM}^{(pq)}$ and $D_{MN}^{(pq)}$, all equal to zero. This fact implies that the TM and TE waves are decoupled. Therefore, TM and TE waves can be treated independently of each other, implying a scalar problem. In this case, the scattering dyadic Green's function $\overline{\overline{G}}_{es}^{(pq)}$ in (17) can be simplified as follows:

$$\begin{aligned}\overline{\overline{G}}_{es}^{(pq)}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_p^2} \times \{&A_{NN}^{(pq)}\boldsymbol{N}_{\substack{e\\o}\eta_p}\boldsymbol{N}'_{\substack{e\\o}\eta_q} \\ &+ A_{MM}^{(pq)}\boldsymbol{M}_{\substack{e\\o}\eta_p}\boldsymbol{M}'_{\substack{e\\o}\eta_q} + B_{NN}^{(pq)}\boldsymbol{N}_{\substack{e\\o}\eta_p}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_q} \\ &+ B_{MM}^{(pq)}\boldsymbol{M}_{\substack{e\\o}\eta_p}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_q} + C_{NN}^{(pq)}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_p}\boldsymbol{N}'_{\substack{e\\o}\eta_q} \\ &+ C_{MM}^{(pq)}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_p}\boldsymbol{M}'_{\substack{e\\o}\eta_q} + D_{NN}^{(pq)}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_p}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_q} \\ &+ D_{MM}^{(pq)}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_p}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_q}\}\end{aligned} \tag{35}$$

where the subscript NN and MM denote the scattering coefficients of TM wave and TE wave, respectively. All the scattering coefficients in (35) can be obtained from (19) to (22) only by taking a set of replacements: $\overline{I} \to 1, \tilde{\overline{R}}_{l,l+1} \to \tilde{R}_{Pl}^E$ or $\tilde{R}_{Pl}^H, \tilde{\overline{R}}_{l,l-1} \to \tilde{R}_{Fl}^E$ or $\tilde{R}_{Fl}^H, \tilde{\overline{T}}_{l,l-1} \to \tilde{T}_{Pl}^E$ or $\tilde{T}_{Pl}^H, \tilde{\overline{T}}_{l,l+1} \to \tilde{T}_{Fl}^E$ or $\tilde{T}_{Fl}^H, \overline{R}_{l,l+1} \to R_{Pl}^E$ or $R_{Pl}^H, \overline{R}_{l,l-1} \to R_{Fl}^E$ or $R_{Fl}^H, \overline{T}_{l,l-1} \to T_{Pl}^E$ or $T_{Pl}^H, \overline{T}_{l,l+1} \to T_{Fl}^E$ or $T_{Fl}^H, (l = 1, 2, \cdots, L)$, where the superscript E and H mean for TM and TE waves, the subscript P and F mean the centri*p*etal and centri*f*ugal reflection or transmission, respectively, and the similar replacements are done from (24) to (33). All the local reflection and transmission coefficients are given by

$$R_{Pp}^H = \frac{\mu_{p+1}\eta_p\mathcal{H}_{pp}\mathcal{H}'_{p+1,p} - \mu_p\eta_{p+1}\mathcal{H}_{p+1,p}\mathcal{H}'_{pp}}{\mu_{p+1}\eta_p\mathcal{H}_{p+1,p}\mathcal{J}'_{pp} - \mu_p\eta_{p+1}\mathcal{J}_{pp}\mathcal{H}'_{p+1,p}} \tag{36}$$

$$R_{Fp}^H = \frac{\mu_{p+1}\eta_p\mathcal{J}_{pp}\mathcal{J}'_{p+1,p} - \mu_p\eta_{p+1}\mathcal{J}_{p1,p}\mathcal{J}'_{pp}}{\mu_{p+1}\eta_p\mathcal{H}_{p+1,p}\mathcal{J}'_{pp} - \mu_p\eta_{p+1}\mathcal{J}_{pp}\mathcal{H}'_{p+1,p}} \tag{37}$$

$$R_{Pp}^E = \frac{\epsilon_{p+1}\eta_p\mathcal{H}_{pp}\mathcal{H}'_{p+1,p} - \epsilon_p\eta_{p+1}\mathcal{H}_{p+1,p}\mathcal{H}'_{pp}}{\epsilon_{p+1}\eta_p\mathcal{H}_{p+1,p}\mathcal{J}'_{pp} - \epsilon_p\eta_{p+1}\mathcal{J}_{pp}\mathcal{H}'_{p+1,p}} \tag{38}$$

$$R_{Fp}^E = \frac{\epsilon_{p+1}\eta_p\mathcal{J}_{pp}\mathcal{J}'_{p+1,p} - \epsilon_p\eta_{p+1}\mathcal{J}_{p+1,p}\mathcal{J}'_{pp}}{\epsilon_{p+1}\eta_p\mathcal{H}_{p+1,p}\mathcal{J}'_{pp} - \epsilon_p\eta_{p+1}\mathcal{J}_{pp}\mathcal{H}'_{p+1,p}} \tag{39}$$

$$T_{Pp}^H = \frac{-2i\mu_p\eta_{p+1}}{(\pi\eta_p a_p)(\mu_{p+1}\eta_p\mathcal{H}_{p+1,p}\mathcal{J}'_{pp} - \mu_p\eta_{p+1}\mathcal{J}_{pp}\mathcal{H}'_{p+1,p})} \tag{40}$$

$$T_{Fp}^H = \frac{-2i\mu_{p+1}\eta_p}{(\pi\eta_{p+1} a_p)(\mu_{p+1}\eta_p\mathcal{H}_{p+1,p}\mathcal{J}'_{pp} - \mu_p\eta_{p+1}\mathcal{J}_{pp}\mathcal{H}'_{p+1,p})} \tag{41}$$

$$T_{Pp}^E = \frac{-2i\epsilon_p\eta_{p+1}}{(\pi\eta_p a_p)(\epsilon_{p+1}\eta_p\mathcal{H}_{p+1,p}\mathcal{J}'_{pp} - \epsilon_p\eta_{p+1}\mathcal{J}_{pp}\mathcal{H}'_{p+1,p})} \tag{42}$$

$$T_{Fp}^E = \frac{2i\epsilon_{p+1}\eta_p}{(\pi\eta_{p+1} a_p)(\epsilon_{p+1}\eta_p\mathcal{H}_{p+1,p}\mathcal{J}'_{pp} - \epsilon_p\eta_{p+1}\mathcal{J}_{pp}\mathcal{H}'_{p+1,p})} \tag{43}$$

where $\mathcal{H}_{kl} = H_0^{(1)}(\eta_k a_l), \mathcal{H}'_{kl} = H_0^{'(1)}(\eta_k a_l), \mathcal{J}_{kl} = J_0(\eta_k a_l), \mathcal{J}'_{kl} = J'_p(\eta_k a_l)$, and $p = 1, 2, \cdots, L$. $\overline{R}_{L,L+1} = \overline{R}_{10} = 0$. Note that the prime operates with respect to the whole variable of the relevant functions. From the group of equations given above, we can easily obtain the scattering coefficients of the dyadic Green's function for the case of axial symmetry.

B. Two Specific Geometries

In order to obtain the specific scattering DGF's of the cylindrically two-, and three-layered media, we only substitute the special values of q and L into (17) and let $p = 1, 2, \cdots, L$, respectively. At the same time, we consider, in particular, the case of axial symmetry ($n = 0$).

1) Cylindrically Two-Layered Media: The geometry of the cylindrically two-layered media can be considered as a single cylinder in an unbounded homogeneous medium. When the current source is located in the different layer of the media, we can obtain a different expression of the dyadic Green's function. Therefore, we should consider two cases of the current source, which is located either in the first layer or in the second layer.

a) Current source located inside the cylinder: In this case, the scattering DGF can be simplified from (17) as follows:

$$\overline{\overline{G}}_{es}^{(11)}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty}\frac{2\delta_0}{\eta_1^2} \times \{[\boldsymbol{N}_{\substack{e\\o}\eta_1}, \boldsymbol{M}_{\substack{e\\o}\eta_1}] \cdot \overline{R}_{12} \cdot [\boldsymbol{N}'_{\substack{e\\o}\eta_1}, \boldsymbol{M}'_{\substack{e\\o}\eta_1}]^T\} \tag{44}$$

$$\overline{\overline{G}}_{es}^{(12)}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty}\frac{2-\delta_0}{\eta_2^2} \times \{[\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2}, \boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2}] \cdot \overline{T}_{12} \cdot [\boldsymbol{N}'_{\substack{e\\o}\eta_1}, \boldsymbol{M}'_{\substack{e\\o}\eta_1}]^T\} \tag{45}$$

where $\overline{R}_{12}$ and $\overline{T}_{12}$ can be given by letting $p = 1$ in (30) and (32) with $\eta_l^2 = k_l^2 - h^2$ and $k_l^2 = \omega^2\mu_l\epsilon_l (l = 1, 2)$.

When $n = 0$, we can simplify (35) as

$$\overline{\overline{G}}_{es}^{(11)}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_1^2} \times \{A_{NN}^{(11)}\boldsymbol{N}_{\substack{e\\o}\eta_1}\boldsymbol{N}'_{\substack{e\\o}\eta_1} + A_{MM}^{(11)}\boldsymbol{M}_{\substack{e\\o}\eta_1}\boldsymbol{M}'_{\substack{e\\o}\eta_1}\} \tag{46}$$

$$\overline{\overline{G}}_{es}^{(21)}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_2^2} \times \{C_{NN}^{(21)}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{N}'_{\substack{e\\o}\eta_1} + C_{MM}^{(21)}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{M}'_{\substack{e\\o}\eta_1}\} \tag{47}$$

where $A_{NN}^{(11)} = R_{P1}^E, A_{MM}^{(11)} = R_{P1}^H, C_{NN}^{(21)} = T_{P1}^E$, and $C_{MM}^{(21)} = T_{P1}^H$. $R_{P1}^{E,H}$ and $T_{P1}^{E,H}$ can be obtained from (36), (38), (40), and (42) by letting $p = 1$.

b) Current source located outside the cylinder: If the source is located outside the cylinder, the scattering DGF is given by

$$\overline{\overline{G}}_{es}^{(12)}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty}\frac{2-\delta_0}{\eta_1^2} \times \{[\boldsymbol{N}_{\substack{e\\o}\eta_1}, \boldsymbol{M}_{\substack{e\\o}\eta_1}] \cdot \overline{T}_{21} \cdot [\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_2}, \boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_2}]^T\} \tag{48}$$

$$\overline{\overline{G}}^{(22)}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \frac{2-\delta_0}{\eta_2^2} \times \{[\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2}, \boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2}] \cdot \overline{R}_{21} \cdot [\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_2}, \boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_2}]^T\} \quad (49)$$

where $\overline{R}_{21}$ and $\overline{T}_{21}$ can be given by letting $p = 1$ in (31) and (33), respectively. When $n = 0$, (35) can be simplified as

$$\overline{\overline{G}}^{(12)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_1^2} \times \{B^{(12)}_{NN}\boldsymbol{N}_{\substack{e\\o}\eta_1}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_2} + B^{(12)}_{MM}\boldsymbol{M}_{\substack{e\\o}\eta_1}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_2}\} \quad (50)$$

$$\overline{\overline{G}}^{(22)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_2^2} \times \{D^{(22)}_{NN}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_2} + D^{(22)}_{MM}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_2}\} \quad (51)$$

where $D^{(22)}_{NN} = R^E_{F1}, D^{(22)}_{MM} = R^H_{F1}, B^{(12)}_{NN} = T^E_{F1}$, and $B^{(12)}_{MM} = T^H_{F1}$. $R^{E,H}_{F1}$ and $T^{E,H}_{F1}$ can be obtained from (37), (39), (41), and (43) by letting $p = 1$.

2) Cylindrically Three-Layered Media: The geometry of the cylindrically three-layered medium is considered a single cylinder with a coating layer superimposed by an unbounded homogeneous medium. The current distribution can be located in any one of the three layers, and the scattering DGF for each layer is presented.

a) Current source lying inside the cylinder: In this case, $q = 1$ and $L = 3$. We can let $p = 1$, 2, or 3 to obtain the scattering DGF in each layer from the general expression (17) of the scattering DFG as follows:

$$\overline{\overline{G}}^{(11)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \frac{2-\delta_0}{\eta_1^2} \times \{[\boldsymbol{N}_{\substack{e\\o}\eta_1}, \boldsymbol{M}_{\substack{e\\o}\eta_1}] \cdot \tilde{\overline{R}}_{12} \cdot [\boldsymbol{N}'_{\substack{e\\o}\eta_1}, \boldsymbol{M}'_{\substack{e\\o}\eta_1}]^T\} \quad (52)$$

$$\overline{\overline{G}}^{(21)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \frac{2-\delta_0}{\eta_2^2} \times \{[\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2}, \boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2}] \cdot \tilde{\overline{R}}_{23} \cdot (\overline{I} - \tilde{\overline{R}}_{21} \cdot \tilde{\overline{R}}_{23})^{-1} \cdot \tilde{\overline{T}}_{12} \cdot [\boldsymbol{N}'_{\substack{e\\o}\eta_1}, \boldsymbol{M}'_{\substack{e\\o}\eta_1}]^T\} \quad (53)$$

$$\overline{\overline{G}}^{(31)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \frac{2-\delta_0}{\eta_3^2} \times \{[\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_3}, \boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_3}] \cdot \tilde{\overline{T}}_{13} \cdot [\boldsymbol{N}'_{\substack{e\\o}\eta_1}, \boldsymbol{M}'_{\substack{e\\o}\eta_1}]^T\} \quad (54)$$

where $\tilde{\overline{R}}_{12}, \tilde{\overline{R}}_{21}, \tilde{\overline{R}}_{23}, \tilde{\overline{T}}_{12}$, and $\tilde{\overline{T}}_{13}$ is given from (24) to (29). When $n = 0$, the scattering DGF's can be simplified as follows:

$$\overline{\overline{G}}^{(11)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_1^2} \times \{A^{(11)}_{NN}\boldsymbol{N}_{\substack{e\\o}\eta_1}\boldsymbol{N}'_{\substack{e\\o}\eta_1} + A^{(11)}_{MM}\boldsymbol{M}_{\substack{e\\o}\eta_1}\boldsymbol{M}'_{\substack{e\\o}\eta_1}\} \quad (55)$$

$$\overline{\overline{G}}^{(21)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_2^2} \times \{A^{(21)}_{NN}\boldsymbol{N}_{\substack{e\\o}\eta_2}\boldsymbol{N}'_{\substack{e\\o}\eta_1} + A^{(21)}_{MM}\boldsymbol{M}_{\substack{e\\o}\eta_2}\boldsymbol{M}'_{\substack{e\\o}\eta_1} + C^{(21)}_{NN}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{N}'_{\substack{e\\o}\eta_1} + C^{(21)}_{MM}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{M}'_{\substack{e\\o}\eta_1}\} \quad (56)$$

$$\overline{\overline{G}}^{(31)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_3^2} \times \{C^{(31)}_{NN}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_3}\boldsymbol{N}'_{\substack{e\\o}\eta_1} + C^{(31)}_{MM}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_3}\boldsymbol{M}'_{\substack{e\\o}\eta_1}\} \quad (57)$$

where

$$A^{(11)}_{NN} = R^E_{P1} + \frac{R^E_{P2}T^E_{P1}T^E_{F1}}{1 - R^E_{F1}R^E_{P2}}, \quad A^{(21)}_{NN} = \frac{R^E_{P2}}{1 - R^E_{F1}R^E_{P2}}$$
$$A^{(11)}_{MM} = R^H_{P1} + \frac{R^H_{P2}T^H_{P1}T^H_{F1}}{1 - R^H_{F1}R^H_{P2}}, \quad A^{(21)}_{MM} = \frac{R^H_{P2}}{1 - R^H_{F1}R^H_{P2}}$$
$$C^{(21)}_{NN} = \frac{T^E_{F1}}{1 - R^E_{F1}R^E_{P2}}, \quad C^{(31)}_{NN} = \frac{T^E_{F1}T^E_{F2}}{1 - R^E_{F1}R^E_{P2}}$$
$$C^{(21)}_{MM} = \frac{T^H_{F1}}{1 - R^H_{F1}R^H_{P2}}, \quad C^{(31)}_{MM} = \frac{T^H_{F1}T^H_{F2}}{1 - R^H_{F1}R^H_{P2}}. \quad (58)$$

b) Current source lying in the coated layer: To save the space, we only give the scattering DGF's for each layer by Letting $n = 0$. In case of $n \neq 0$, the scattering DGF's can be obtained by following a similar procedure and by carrying out some more repeat derivation. From (35), letting $p = 1$, 2, or 3 yields

$$\overline{\overline{G}}^{(12)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_1^2} \times \{A^{(12)}_{NN}\boldsymbol{N}_{\substack{e\\o}\eta_1}\boldsymbol{N}'_{\substack{e\\o}\eta_2} + A^{(12)}_{MM}\boldsymbol{M}_{\substack{e\\o}\eta_1}\boldsymbol{M}'_{\substack{e\\o}\eta_2} + B^{(12)}_{NN}\boldsymbol{N}_{\substack{e\\o}\eta_1}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_2} + B^{(12)}_{MM}\boldsymbol{M}_{\substack{e\\o}\eta_1}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_2}\} \quad (59)$$

$$\overline{\overline{G}}^{(22)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_2^2} \times \{A^{(22)}_{NN}\boldsymbol{N}_{\substack{e\\o}\eta_2}\boldsymbol{N}'_{\substack{e\\o}\eta_2} + A^{(22)}_{MM}\boldsymbol{M}_{\substack{e\\o}\eta_2}\boldsymbol{M}'_{\substack{e\\o}\eta_2} + B^{(22)}_{NN}\boldsymbol{N}_{\substack{e\\o}\eta_2}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_2} + B^{(22)}_{MM}\boldsymbol{M}_{\substack{e\\o}\eta_2}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_2} + C^{(22)}_{NN}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{N}'_{\substack{e\\o}\eta_2} + C^{(22)}_{MM}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{M}'_{\substack{e\\o}\eta_2} + D^{(22)}_{NN}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_2} + D^{(22)}_{MM}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_2}\} \quad (60)$$

$$\overline{\overline{G}}^{(32)}_{es}(\overline{R},\overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2-\delta_0}{\eta_3^2} \times \{C^{(32)}_{NN}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_3}\boldsymbol{N}'_{\substack{e\\o}\eta_2} + C^{(32)}_{MM}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_3}\boldsymbol{M}'_{\substack{e\\o}\eta_2} + D^{(32)}_{NN}\boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_3}\boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_2} + D^{(32)}_{MM}\boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_3}\boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_2}\} \quad (61)$$

where

$$A^{(12)}_{NN} = \frac{R^E_{P2}T^E_{P2}T^E_{P1}}{1 - R^E_{F1}R^E_{P2}}, \quad A^{(12)}_{MM} = \frac{R^H_{P2}T^H_{P2}T^H_{P1}}{1 - R^H_{F1}R^H_{P2}}$$
$$B^{(12)}_{NN} = \frac{T^E_{P1}}{1 - R^E_{F1}R^E_{P2}}, \quad B^{(12)}_{MM} = \frac{T^H_{P1}}{1 - R^H_{F1}R^H_{P2}}$$
$$A^{(22)}_{NN} = \frac{R^E_{P2}}{1 - R^E_{F1}R^E_{P2}}, \quad A^{(22)}_{MM} = \frac{R^H_{P2}}{1 - R^H_{F1}R^H_{P2}}$$
$$B^{(22)}_{NN} = \frac{R^E_{F1}R^E_{P2}}{1 - R^E_{F1}R^E_{P2}}, \quad B^{(22)}_{MM} = \frac{R^H_{F1}R^H_{P2}}{1 - R^H_{F1}R^H_{P2}}$$
$$C^{(22)}_{NN} = \frac{R^E_{F1}R^E_{P2}}{1 - R^E_{F1}R^E_{P2}}, \quad C^{(22)}_{MM} = \frac{R^H_{F1}R^H_{P2}}{1 - R^H_{F1}R^H_{P2}}$$
$$D^{(22)}_{NN} = \frac{R^E_{F1}}{1 - R^E_{F1}R^E_{P2}}, \quad D^{(22)}_{MM} = \frac{R^H_{F1}}{1 - R^H_{F1}R^H_{P2}}$$

$$C_{NN}^{(32)} = \frac{T_{F2}^E}{1 - R_{F1}^E R_{P2}^E}, \quad C_{MM}^{(32)} = \frac{T_{F2}^H}{1 - R_{F1}^H R_{P2}^H}$$
$$D_{NN}^{(32)} = \frac{T_{F2}^E R_{F1}^E}{1 - R_{F1}^E R_{P2}^E}, \quad D_{MM}^{(32)} = \frac{T_{F2}^H R_{F1}^H}{1 - R_{F1}^H R_{P2}^H}. \tag{62}$$

c) Current source lying outside the coated layer: When the source is located outside the coated layer, the scattering DGF's for $n = 0$ is given by

$$\overline{\overline{G}}_{es}^{(13)}(\overline{R}, \overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2 - \delta_0}{\eta_1^2} \times \{B_{NN}^{(13)} \boldsymbol{N}_{\substack{e\\o}\eta_1} \boldsymbol{N}'_{\substack{e\\o}\eta_3} + B_{MM}^{(13)} \boldsymbol{M}_{\substack{e\\o}\eta_1} \boldsymbol{M}'_{\substack{e\\o}\eta_3}\} \tag{63}$$

$$\overline{\overline{G}}_{es}^{(23)}(\overline{R}, \overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2 - \delta_0}{\eta_2^2} \times \{B_{NN}^{(23)} \boldsymbol{N}_{\substack{e\\o}\eta_2} \boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_3} + B_{MM}^{(23)} \boldsymbol{M}_{\substack{e\\o}\eta_2} \boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_3} + D_{NN}^{(23)} \boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_2} \boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_3} + D_{MM}^{(23)} \boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_2} \boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_3}\} \tag{64}$$

$$\overline{\overline{G}}_{es}^{(33)}(\overline{R}, \overline{R}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dh \frac{2 - \delta_0}{\eta_3^2} \times \{D_{NN}^{(33)} \boldsymbol{N}^{(1)}_{\substack{e\\o}\eta_3} \boldsymbol{N}'^{(1)}_{\substack{e\\o}\eta_3} + D_{MM}^{(33)} \boldsymbol{M}^{(1)}_{\substack{e\\o}\eta_3} \boldsymbol{M}'^{(1)}_{\substack{e\\o}\eta_3}\} \tag{65}$$

where

$$B_{NN}^{(13)} = \frac{T_{P2}^E T_{P1}^E}{1 - R_{F1}^E R_{P2}^E}, \quad B_{NN}^{(23)} = \frac{T_{P2}^E}{(1 - R_{F1}^E R_{P2}^E)^2}$$
$$B_{MM}^{(13)} = \frac{T_{P2}^H T_{P1}^H}{1 - R_{F1}^H R_{P2}^H}, \quad B_{MM}^{(23)} = \frac{T_{P2}^H}{(1 - R_{F1}^H R_{P2}^H)^2}$$
$$D_{NN}^{(23)} = \frac{R_{F1}^E T_{P2}^E}{(1 - R_{F1}^E R_{P2}^E)^2}, \quad D_{NN}^{(33)} = R_{F2}^E + \frac{R_{F1}^E T_{F2}^E T_{P2}^E}{1 - R_{F1}^E R_{P2}^E}$$
$$D_{MM}^{(23)} = \frac{R_{F1}^H T_{P2}^H}{(1 - R_{F1}^H R_{P2}^H)^2}, \quad D_{MM}^{(33)} = R_{F2}^H + \frac{R_{F1}^H T_{F2}^H T_{P2}^H}{1 - R_{F1}^H R_{P2}^H}. \tag{66}$$

Up to now, all the scattering DGF's for several specific cases have been given in detail. The results for the specific cases agree with those given by other researchers, showing the validity of our new expression of DGF. Our expression, however, is more general and can be applied to more complicated problems with any number of cylindrically multilayered lossless or lossy media.

V. Conclusion

As a very powerful and elegant method for solving the boundary-value problems, the dyadic Green's function technique has been well developed and employed for more than two decades. As mentioned above, the dyadic Green's function for the planarly stratified media has been well investigated by many researchers, and the general expressions of those functions have been given in detail. The DGF's for the spherically stratified media have also been investigated, and the general expressions have been presented recently. Although several simple geometries of the cylindrically stratified media extracted from practical engineering problems have been investigated, it is very useful to construct the general formulation of dyadic Green's functions for arbitrary multilayered cylindrical media.

In this paper, the general expression of the DGF for cylindrically multilayered media has been derived. Some concepts, such as the effective reflection and transmission of plane waves, are used and extended to derive the scattering coefficient matrices. Several specific cases, i.e., axial symmetry ($n = 0$), two-layered and three-layered media, are considered in detail. Because of the duality and the superposition principle, the magnetic type of DGF's can been derived easily by the replacements $\boldsymbol{E} \rightarrow \boldsymbol{H}, \boldsymbol{H} \rightarrow -\boldsymbol{E}, \boldsymbol{J} \rightarrow \boldsymbol{M}, \boldsymbol{M} \rightarrow -\boldsymbol{J}, \mu \rightarrow \epsilon, \epsilon \rightarrow \mu$. Therefore, only the electric type of dyadic Green's functions are analyzed. In order to check the general formulation we have derived, the scattering DGF's for several special cases have been obtained by simplifying the general expression of DGF's. The agreement between the results for the specific cases derived in this paper and the ones given by other researchers has demonstrated the validity of our approach. However, our expression is more general and can be applied to much more complicated problems. Our new expression is more general and can be applied to problems with any number of cylindrically multilayered lossless or lossy media.

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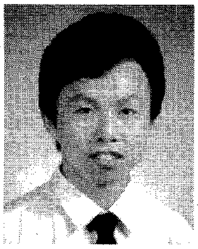

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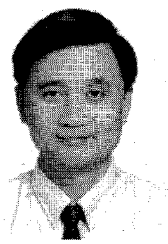

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